Right-Value

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1. Introduction

It is well known that the implied volatilities of quoted European options are non-constant and depend both on the maturity and strike of the option, a phenomenon often referred to as the "smile". Several problems arise in the presence of smile. First, arbitrage may exist among the quoted options. Another problem is to price European options for strikes and maturities not quoted on the market. Standard interpolation techniques may give rise to arbitrage in the interpolated volatilities surface even if there is no arbitrage in the original set. A related problem is to price non-vanilla options, such as barrier options, by taking the smile into account.

2. Right-Value

Our software Right-Value deals with smile-related problems in the index, equity and forex markets. The single-maturity interpolation method of Right-Value can be applied to the interest rate market as well. Right-Value contains three components. The first component deals with arbitrage. Right-Value exhibits an arbitrage opportunity among input prices if and only if there is any. If no arbitrage is found, Right-Value computes the largest volatility interval for any strike and maturity entered by the user that induces no arbitrage with the input volatilities. The second component deals with interpolation. If the market volatilities are arbitrage-free Right-Value computes an interpolating surface of the market volatilities for all strikes and maturities up to the last maturity that is arbitrage-free and satisfies some smoothness conditions. The interpolated volatilities can be used to calibrate Dupire's model [2], according to which the spot follows the following stochastic differential equation:

$$dS_t = \mu_t S_t \, dt + \sigma_{loc}(S_t,t) \, S_t \, dW_t$$

where W_t is a Brownian motion and $\sigma_{loc}(S_t,t)$ is a deterministic function. Dupire has shown that the local volatilities can be computed using the following formula (which we state for simplicity in the case where there are no dividends and no interest rates)

$$\sigma_{loc}{}^2(K,t) = \frac{2\partial C(K,t)/\partial t}{K^2 \partial^2 C(K,t)/\partial K^2}$$

where C(K,T) is today's price of a European call with strike K and maturity T. The local volatilities surface can be used to price exotic options by taking the smile into account. The third component of Right-Value prices knock-out barrier options using this approach. Local volatilities computation depends strongly on the interpolation method used. Our interpolation, unlike standard interpolation methods, leads to stable prices in practice.

3. Our Interpolation method

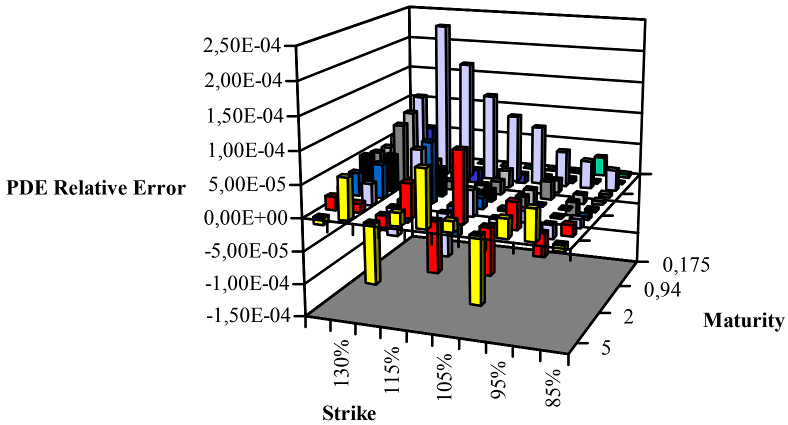

Fig. 1: Relative errors on call prices of S&P 500 on Oct 95, using Crank-Nicholson's PDE method and Right-Value local volatilities surface, with 100 time-steps and 1000 space steps.

The basis block for our interpolation is a single-maturity interpolation with the following properties (the first one holds only if there are no absolute discrete dividends):

- If the input volatilities are constant so are the interpolated volatilities.
- The second derivative of the call price with respect to the strike is positive and continuous.

The motivation for the second property is that the second derivative of the call price is proportional to the implied density of the spot. Our single-maturity interpolation method has the following properties:

- Robustness: our interpolation does not depend on the shape of the discrete volatilities and applies to index, equity, forex and interest rate options.
- Accuracy: the quality of the fit is excellent.
- Speed: it takes seconds to calibrate a 10×10 volatility matrix on an 800 Mhz processor.

4. Numerical Results

We show below the results of tests we did on the implied volatility matrix of the

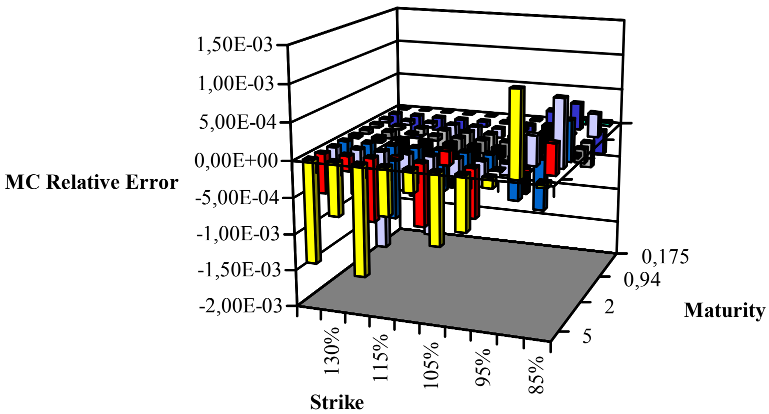

Fig. 2: Relative errors on call prices of S&P 500 on Oct 95, using a standard Monte Carlo method and Right-Value local volatilities surface, with 100 time-steps and 10^5 samples.

S&P 500 index on October 1995, given in [1]. The spot is $590, the interest rate is r = 6% and the dividend yield is q = 2.62%. The implied volatility matrix is given by:

K % \T	0,175	0,425	0,695	0,94	1	1,5	2	3	4	5
85	19,00	17,70	17,20	17,10	17,10	16,90	16,90	16,80	16,80	16,80
90	16,80	15,50	15,70	15,90	15,90	16,00	16,10	16,10	16,20	16,40
95	13,30	13,80	14,40	14,90	15,00	15,10	15,30	15,50	15,70	15,90
100	11,30	12,50	13,30	13,70	13,80	14,20	14,50	14,90	15,20	15,40
105	10,20	10,90	11,80	12,70	12,80	13,30	13,70	14,30	14,80	15,10
110	9,70	10,30	10,40	11,30	11,50	12,40	13,00	13,70	14,30	14,80
115	12,00	10,00	10,00	10,60	10,70	11,90	12,60	13,30	13,90	14,40
120	14,20	11,40	10,10	10,30	10,30	11,30	11,90	12,80	13,50	14,00
130	16,90	13,00	10,80	10,00	9,90	10,70	11,50	12,40	13,00	13,60
140	20,00	15,00	12,40	11,00	10,80	10,20	11,10	12,30	12,80	13,20

We computed the prices of European calls using the local volatilities surface together with PDE (Crank-Nicholson) and Monte Carlo methods. Fig. 1 and 2 show the relative errors obtained. By definition the relative error is the difference between the price computed using the local volatilities surface and the one computed from the input implied volatility using the Black-and-Scholes formula, divided by the spot.

5. Conclusion

Right-Value deals with smile-related problems such as arbitrage and interpolation of implied volatilities. The local volatilities surface generated by Right-Value can be used to compute stable prices of exotic options.

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